

MATEMATIKA 1: KOLOKVIJ 1: Trajanje 100 minuta. Zabranjen je razgovor sa drugim studentima. Na klupama je dozvoljen samo pisaći pribor, kalkulator, indeks ili iksica i prazni papiri koji nose ime studenta. Sav ostali pribor, formule, uređaji, bilješke i nepotpisane prazne papire zabranjeno je koristiti i trebaju ostati u torbi ili pohranjeni kod nastavnika (elektronički uređaji trebaju biti isključeni) tokom cijelog trajanja ispita. Studenti koji primijete zabranjene predmete dužni su ih prijaviti nastavniku. Nije dozvoljeno međusobno posuđivanje pribora tijekom trajanja ispita. Povreda ovih pravila može za posljedicu imati udaljavanje s ispita. ZADATKE RIJEŠAVATE JEDNOSTRANO NA PREDLOŠKU KOJI MOŽETE DOBITI OD NASTAVNIKA.

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Broj ↓
bodova

IME I PREZIME: HRVOJE BATUR

BROJ INDEKSA: 17-2-0006-2010

1. Riješiti jednačinu: $1 - i = z^4 - (-i)^{113}$.

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2. Odrediti inverz i determinantu matrice:

$$A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 2 \\ -3 & 1 & 3 \end{bmatrix}$$

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Izračunati matrični umnožak AA^{-1} .

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3. Za funkciju arkus tangens nacrtati graf i navesti: domenu, kodomenu, periodičnost, (ne)parnost, ograničenost, rast ili pad; da li je injekcija, surjekcija ili bijekcija; da li postoji inverz i ako postoji koja je to funkcija.

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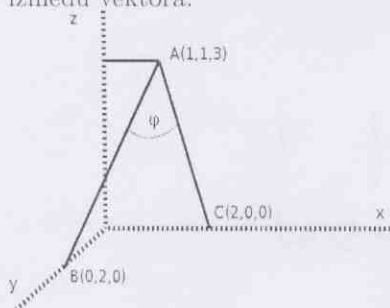
4. Gaussovom metodom riješiti matrični sustav:

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$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 3 \\ 1 & 1 & 3 & 3 \\ 1 & 3 & 3 & 3 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \\ 2 \end{bmatrix}$$

5. Zadana je konfiguracija nosača kao na slici ispod. Potrebno je odrediti kut φ korištenjem formule za kut između vektora.

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ODLIČAN REZULTAT!
MOLIM UBUDUĆE PIŠITE
JEDNOSTRANO.

ZADATKE RIJEŠAVATE JEDNOSTRANO NA OVOM PAPIRU, ALI NA DRUGOJ STRANI. NA OVOJ STRANI MOŽETE PISATI, ALI SVE ŠTO OVDJE NAPIŠETE NEĆE VAM BITI PREGLEDANO NITI OCIJENJENO.

$$\left[\begin{array}{ccc|ccc} 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 1 & 0 \\ -3 & 1 & 3 & 0 & 0 & 1 \end{array} \right] \begin{array}{l} ||-2I \\ \cdot(-\frac{1}{3}) \end{array} \sim \left[\begin{array}{ccc|ccc} 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & -2 & 1 & 0 \\ 1 & -1/3 & -1 & 0 & 0 & -1/3 \end{array} \right] \begin{array}{l} ||I+1 \end{array}$$

$$\left[\begin{array}{ccc|ccc} 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & -2 & 1 & 0 \\ 1 & -1/3 & 0 & 1 & 0 & -1/3 \end{array} \right] \begin{array}{l} ||I+1/3I \end{array} \sim \left[\begin{array}{ccc|ccc} 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & -2 & 1 & 0 \\ 1 & 0 & 0 & 1/3 & 1/3 & -1/3 \end{array} \right]$$

$$A \cdot A^{-1} = \left[\begin{array}{ccc|ccc} 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & -2 & 1 & 0 \\ -3 & 1 & 3 & 1/3 & 1/3 & -1/3 \end{array} \right] \begin{array}{l} 1-2/3 \\ \cdot \\ 3 \end{array}$$



$$\left[\begin{array}{ccc} 1/3 & 1+2/3 & -2/3 \\ 1/3 & 1+2/3 & -1 \\ -3-2+1 & 1+1 & -1 \end{array} \right]$$

$$\left[\begin{array}{ccc} 1/3 & 5/3 & -2/3 \\ 1/3 & 5/3 & -1 \\ 4 & 2 & -1 \end{array} \right]$$

$$\left[\begin{array}{ccc|ccc} 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & -2 & 1 & 0 \\ -3 & 1 & 3 & 1/3 & 1/3 & -5/3 \end{array} \right]$$

$$\left[\begin{array}{ccc} 1/3 & 1/3 & -5/3 \\ 2/3 & 1+2/3 & -2-10/3 \\ 1 & 1+1 & -2-5 \end{array} \right]$$

$$\left[\begin{array}{ccc} 1/3 & 1/3 & -5/3 \\ 2/3 & 5/3 & -16/3 \\ 1 & 2 & -7 \end{array} \right]$$

① $1-i = z^4 - (-i)^{1/3}$

$-i^{1/3} = -i^1 = -i$

$1-i = z^4 + i$

$1+i = z^4 + i$

$1+i-i = z^4$

$z^4 = 1 \quad \sqrt[4]{\quad}$

$z = \sqrt[4]{1}$

$r = 1$ ✓

$\varphi = 0$ ✓

$z_1 = 1 \left(\cos \frac{\varphi}{4} + i \sin \frac{\varphi}{4} \right)$

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$z_1 = 1 \left(\cos 0 + i \sin 0 \right)$ ✓

$z_1 = 1$ ✓

$z_2 = 1 \left(\cos \frac{2\pi}{4} + i \sin \frac{2\pi}{4} \right)$ ✓

$z_3 = 1 \left(\cos \frac{4\pi}{4} + i \sin \frac{4\pi}{4} \right)$

$= 1 \left(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2} \right)$

$z_3 = 1 \left(\cos \pi + i \sin \pi \right)$

$= 0 + i$

$z_3 = -1$ ✓

$z_2 = i$ ✓

4.71238

$z_4 = 1 \left(\cos \frac{6\pi}{4} + i \sin \frac{6\pi}{4} \right)$

$z_4 = 1 \left(\cos \frac{3\pi}{2} + i \sin \frac{3\pi}{2} \right)$

$z_4 = 0 - i$ ✓

$z_4 = -i$

$\det(A) = \begin{vmatrix} 0 & 0 & 1 \\ 0 & 1 & 2 \\ -3 & -1 & 3 \end{vmatrix} = +1 \begin{vmatrix} 0 & 1 \\ -3 & 1 \end{vmatrix}$
 $= 1(0 - (-3))$
 $= 1 \cdot 3$
 $= 3$ ✓

$A^{-1} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & -2 \\ 1/3 & 1/3 & -5/3 \end{bmatrix}$? X

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② $A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 2 \\ -3 & 1 & 3 \end{bmatrix}$

$AA^{-1} = \begin{bmatrix} 1/3 & 1/3 & -5/3 \\ 2/3 & 5/3 & -16/3 \\ 1 & 2 & -7 \end{bmatrix}$

VIDI MUŽANović

TREBA ISPASTI

JEDINIČNA MATRICA
JER UVIJEK VRIJEDI

$AA^{-1} = I \quad ; \quad A^{-1}A = I$

ZADATKE RIJEŠAVATE JEDNOSTRANO NA OVOM PAPIRU, ALI NA DRUGOJ STRANI. NA OVOJ STRANI MOŽETE PISATI, ALI SVE ŠTO OVDJE NAPIŠETE NEĆE VAM BITI PREGLEDANO NITI OCIJENJENO.

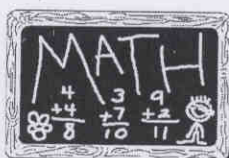
$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 2 \\ -3 & 1 & 3 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & -2 \\ 1/3 & 0 & 1/3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 2 \\ -3 & 1 & 3 \end{bmatrix} \begin{bmatrix} 1/3 & 1/3 & -1/3 \\ -2 & 1 & 0 \\ 2/3 & -1/3 & 1/3 \end{bmatrix}$$

$2/3 \quad 1-2/3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ +3 & 1 & -3 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & -2 \\ 1/3 & 0 & 1/3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 1 & 0 \\ -3 & 1 & 3 & 0 & 0 & 1 \end{array} \right] \sim \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 2 & 1 & 2 & 0 & 1 & 0 \\ 0 & 1 & 3 & 0 & 0 & 1 \end{array} \right] \begin{array}{l} \\ \parallel - 2I \\ \\ \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & -2 & 1 & 0 \\ 0 & 1 & 3 & 0 & 0 & 1 \end{array} \right] \text{III} - \text{II}$$



$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & -2 & 1 & 0 \\ 0 & 0 & 3 & 2 & -1 & 1 \end{array} \right] \begin{array}{l} \\ \\ /:3 \end{array} \sim \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & -2 & 1 & 0 \\ 0 & 0 & 1 & 2/3 & -1/3 & 1/3 \end{array} \right] \begin{array}{l} \\ \\ I - \text{III} \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/3 & 1/3 & -1/3 \\ 0 & 1 & 0 & -2 & 1 & 0 \\ 0 & 0 & 1 & 2/3 & -1/3 & 1/3 \end{array} \right]$$

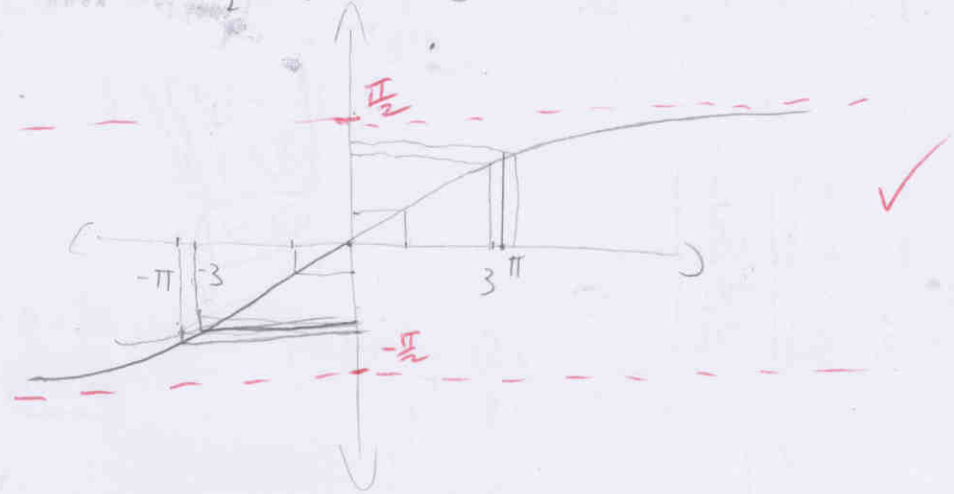
4)
$$\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 1/2 \\ 1/2 \\ -1/2 \\ 1/2 \end{bmatrix}$$

$\frac{1}{2} + \frac{1}{2} - \frac{1}{2} + \frac{1}{2} = 1 \Rightarrow \frac{2}{2} = 1 \Rightarrow 1 = 1$
 $\frac{1}{2} + \frac{1}{2} - \frac{1}{2} + \frac{3}{2} = 2 \Rightarrow \frac{4}{2} = 2 \Rightarrow 2 = 2$
 $\frac{1}{2} + \frac{1}{2} - \frac{3}{2} + \frac{3}{2} = 1 \Rightarrow \frac{2}{2} = 1 \Rightarrow 1 = 1$
 $\frac{1}{2} + \frac{3}{2} - \frac{3}{2} + \frac{3}{2} = 2 \Rightarrow \frac{4}{2} = 2 \Rightarrow 2 = 2$

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3)

x	f(x)
$-\pi$	-1.26
-3	-1.24
-1	-0.78
0	0
1	0.78
3	1.24
π	1.26



$DA = [-\frac{\pi}{2}, \frac{\pi}{2}]$ X

-> funkcija nije periodična ✓

$KA = [-1, 1]$ X

-> $f(-x) = -f(x)$ neparna ✓

$f(-3) = -f(3)$
 $-1.24 = -1.24$

$D(f) = \mathbb{R}$

-> funkcija raste ✓

kodomena:

-> funkcija je ograničena od $-\frac{\pi}{2}$ do $\frac{\pi}{2}$ ✓

$f(x) \in \langle -\frac{\pi}{2}, \frac{\pi}{2} \rangle$

-> funkcija je surjektivna, injektivna i bijektivna (ako je bijektivna ima inverz) ✓

-> funkcija ima inverz i to je funkcija $f(x) = \tan(x)$, NA

DOMENI $\langle -\frac{\pi}{2}, \frac{\pi}{2} \rangle$

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ZADATKE RIJEŠAVATE JEDNOSTRANO NA OVOM PAPIRU, ALI NA DRUGOJ STRANI. NA OVOJ STRANI MOŽETE PISATI, ALI SVE ŠTO OVDJE NAPIŠETE NEĆE VAM BITI PREGLEDANO NITI OCIJENJENO.

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 3 & 2 \\ 1 & 1 & 3 & 3 & 1 \\ 1 & 3 & 3 & 3 & 2 \end{array} \right] \begin{array}{l} \text{II}-\text{I} \\ \text{III}-\text{I} \\ \text{IV}-\text{I} \end{array} \sim \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 2 & 1 \\ 0 & 0 & 2 & 2 & 0 \\ 0 & 2 & 2 & 2 & 1 \end{array} \right] \begin{array}{l} 2d=1 \\ \text{III} \cdot \frac{1}{2} \quad d=\frac{1}{2} \\ \text{IV}-\text{II} \quad c+2d=0 \end{array}$$

$$2c+1=0$$

$$2c=-1$$

$$c = -\frac{1}{2}$$

$$\begin{array}{l} 2b-1+1=2 \\ 2b=2 \\ b=1 \end{array}$$

$$\begin{array}{l} a+1-\frac{1}{2}+\frac{1}{2}=1 \\ a=0 \end{array}$$

$$2b-1+1=1$$

$$2b=1$$

$$b = \frac{1}{2}$$

$$2c+2d=0$$

$$2c+1=0$$

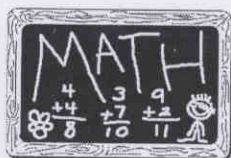
$$c = -\frac{1}{2}$$

$$a + \frac{1}{2} - \frac{1}{2} + \frac{1}{2} = 1$$

$$a = 1 - \frac{1}{2}$$

$$a = \frac{1}{2}$$

$$\frac{1}{2} + \frac{1}{2} - \frac{1}{2} + \frac{1}{2}$$



$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 2 & 1 \\ 0 & 0 & 2 & 0 & -1 \\ 0 & 2 & 0 & 0 & 1 \end{array} \right] \begin{array}{l} :2 \\ :2 \\ :2 \end{array}$$

$$2d=1$$

$$d = \frac{1}{2}$$

$$2c=-1$$

$$c = -\frac{1}{2}$$

$$2b=1 \quad \left[\begin{array}{cccc|c} 1 & 0 & 1 & 1 & 1/2 \\ 0 & 0 & 0 & 1 & 1/2 \\ 0 & 0 & 1 & 0 & -1/2 \\ 0 & 1 & 0 & 0 & 1/2 \end{array} \right] \begin{array}{l} \text{I}-\text{II} \\ \\ \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1/2 \\ 0 & 0 & 1 & 0 & -1/2 \\ 0 & 1 & 0 & 0 & 1/2 \end{array} \right] \begin{array}{l} \text{I}-\text{III} \\ \\ \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1/2 \\ 0 & 0 & 1 & 0 & -1/2 \\ 0 & 1 & 0 & 0 & 1/2 \end{array} \right] \begin{array}{l} \text{I}-\text{IV} \\ \\ \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 1/2 \\ 0 & 0 & 0 & 1 & 1/2 \\ 0 & 0 & 1 & 0 & -1/2 \\ 0 & 1 & 0 & 0 & 1/2 \end{array} \right]$$

IME I PREZIME: HRVOJE BATUR

BROJ INDEKSA: 17-2-0006-2010

5) $A = (1, 1, 3)$
 $B = (0, 2, 0)$
 $C = (2, 0, 0)$

$$\vec{AB} = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix} \checkmark$$

$$\vec{AC} = \begin{bmatrix} 1 \\ -1 \\ -3 \end{bmatrix} \checkmark$$

$$\vec{AB} \times \vec{AC} = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix} \times \begin{bmatrix} 1 \\ -1 \\ -3 \end{bmatrix} = \begin{bmatrix} -3-3 \\ -3-3 \\ +1-1 \end{bmatrix} = \begin{bmatrix} -6 \\ -6 \\ 0 \end{bmatrix} \checkmark$$

$$\|\vec{AB} \times \vec{AC}\| = \sqrt{36+36} = \sqrt{72} = 8.48528 \checkmark$$

$$\|\vec{AB}\| = \sqrt{1+1+9} = \sqrt{11} = 3.3166 \checkmark$$

$$\|\vec{AC}\| = \sqrt{1+1+0} = \sqrt{2} = 1.4142 \checkmark$$

$$\|\vec{AB} \times \vec{AC}\| = \|\vec{AB}\| \|\vec{AC}\| \sin \angle(\vec{AB}, \vec{AC})$$

$$8.848528 = 3.3166 \cdot 3.3166 \sin \angle(\vec{AB}, \vec{AC})$$

$$\sin \angle(\vec{AB}, \vec{AC}) = \frac{8.848528}{10.999835} = 0.8044237$$

$$\angle(\vec{AB}, \vec{AC}) = 0.9347$$

20 0.594

MOŽDA JE LAKŠE PREKO: $\vec{AB} \cdot \vec{AC} = \|\vec{AB}\| \cdot \|\vec{AC}\| \cdot \cos \angle(\vec{AB}, \vec{AC})$

VIDI MUŽANOVIC
JURLINA

VIDI RAZLIKU U ODNOSU NA NJIHOV REZULTAT
 $\angle(\vec{AB}, \vec{AC}) = 0.88$

ŠTO JE OKO 6% RAZLIKE

ZADATKE RIJEŠAVATE JEDNOSTRANO NA OVOM PAPIRU, ALI NA DRUGOJ STRANI. NA OVOJ STRANI MOŽETE PISATI, ALI SVE ŠTO OVDJE NAPIŠETE NEĆE VAM BITI PREGLEDANO NITI OCIJENJENO.

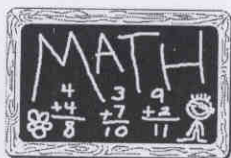
$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 3 & 2 \\ 1 & 1 & 3 & 3 & 1 \\ 1 & 3 & 3 & 3 & 2 \end{array} \right] \begin{array}{l} \text{II}-\text{I} \\ \text{III}-\text{I} \\ \text{IV}-\text{I} \end{array} \sim \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 1 \\ -2 & -2 & -2 & 0 & -1 \\ -2 & -2 & 0 & 0 & -2 \\ -2 & 0 & 0 & 0 & -1 \end{array} \right] \begin{array}{l} \\ \\ :(-2) \end{array} \sim \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 1 \\ -2 & -2 & -2 & 0 & -1 \\ -2 & -2 & 0 & 0 & -2 \\ 1 & 0 & 0 & 0 & 1/2 \end{array} \right] \begin{array}{l} \\ :(-2) \\ \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1/2 \\ 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1/2 \end{array} \right] \begin{array}{l} \text{I}-\text{II} \\ \text{II}-\text{III} \\ \text{III}-\text{IV} \end{array} \sim \left[\begin{array}{cccc|c} 0 & 0 & 0 & 1 & 1/2 \\ 0 & 0 & 1 & 0 & -1/2 \\ 0 & 1 & 0 & 0 & 1/2 \\ 1 & 0 & 0 & 0 & 1/2 \end{array} \right]$$

$$\frac{1}{2} - \frac{1}{2} \quad \frac{1}{2} + \frac{1}{2} = 1$$

$$\frac{1}{2} - \frac{1}{2} + \frac{1}{2} + \frac{3}{2} = 2$$

$$\frac{1}{2} + \frac{1}{2} - \frac{1}{2} + \frac{1}{2} = 1$$



$$\frac{1}{2} - \frac{1}{2} + \frac{3}{2} + \frac{3}{2}$$

$$\frac{1}{2} + \frac{1}{2} - \frac{1}{2} + \frac{3}{2} = 2$$

$$\left[\begin{array}{ccc|ccc} 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 2 & 0 & 1 & 0 \\ -3 & 1 & 3 & 1 & 0 & 0 \end{array} \right] \begin{array}{l} \\ \text{I}-2\text{II} \\ \end{array}$$

$$\frac{1}{2} + \frac{1}{2} - \frac{2}{2} + \frac{3}{2} = 1$$

$$\left[\begin{array}{ccc|ccc} 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & -2 \\ -3 & 1 & 3 & 1 & 0 & 0 \end{array} \right] \begin{array}{l} \\ \\ :(-3) \end{array}$$

$$\frac{1}{2} + \frac{3}{2} - \frac{3}{2}$$

$$\left[\begin{array}{ccc|ccc} 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & -2 \\ 1 & -1/3 & 1 & 1/3 & 0 & 0 \end{array} \right] \begin{array}{l} \\ \\ \text{III}-\text{I} \end{array}$$

$$\left[\begin{array}{ccc|ccc} 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & -2 \\ 1 & -1/3 & 0 & 1/3 & 0 & -1 \end{array} \right] \begin{array}{l} \\ \\ \text{III}+1/3\text{II} \end{array} \sim \left[\begin{array}{ccc|ccc} 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & -2 \\ 1 & 0 & 0 & 1/3 & 1/3 & -5/3 \end{array} \right]$$

$$-1 - \frac{2}{3} = \frac{-3-2}{3}$$