MATEMATIKA 3: Ispit traje 2 sata. Na klupama je dozvoljen samo pisaći pribor, kalkulator, indeks ili iksica i tablice Laplaceovil transformacija. Sav ostali pribor, formule, uređaji i bilješke zabranjeno je koristiti i trebaju ostati u torbi ili pohranjeni kod nastavnika (elektronički uređaji trebaju biti isključeni) tokom cijelog trajanja ispita. Studenti koji primijete zabranjene predmete dužni su ih prijaviti nastavniku. Svim studentima u neposrednoj blizini zabranjenih predmeta prijeti isključenje s ispita

ZADATKE RIJEŠAVATE NA OVAJ PAPIR.

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1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu:

$$y'''(t) + 3y'(t) = t^2, \quad y(0)' = y''(0) = 0, \; y(0) = 1.$$

2. Izračunajte površinu oplošja paraboloida $x^2 + y^2 = 4z$, $z \leq 4$.

3. Izračunati $\int_{\widehat{ABC}} z^2 dx + y^2 dy + x dz$ gdje je $\widehat{ABC}$ krivulja koja ide bridovima trokuta s vrhovima $A(1,0,0)$, $B(0,1,0)$ $C(0,0,0)$ usmjerena redom od vrha A preko B i C do ponovo vrha A. Koristiti Stokesovu formulu.

4. Izračunati integral funkcije $f(x,y,z) = x$ u dijelu prostora omeđenog plohama $x = z^2$, $z = x$, $y = -1$ i $y = 1$.

5. Neka je S područje zadano nejednadžbom $|x| + |y| \leq 1$. Izračunati dvostruki integral

$$\iint_S xy \, dx dy$$

1.) $y'''(t) + 3y'(t) = t^2 \qquad y'(0) = y''(0) = 0, \; y(0) = 1$

$$L[y'''(t)] + 3L[y'(t)] = L[t^2]$$

$$\left(s^3 Y(s) - s^{2}\underbrace{y(0)}_{=1} - s\underbrace{y'(0)}_{=0} - \underbrace{y''(0)}_{=0}\right) + 3\cdot\left(sY(s) - y(0)\right) = \frac{2!}{s^3}$$

$$s^3 Y(s) - s + 3sY(s) - 3 = \frac{2!}{s^3}$$

$$s^3 Y(s) - 3sY(s) = \frac{2!}{s^3} + s + 3$$

$$s^3 - 3s = s(s^2 - 3)$$

$$Y(s)(s^3 - 3s) = \frac{2}{s^3} + s + 3 \qquad /:(s^3 - 3s)$$

$$Y(s) = \frac{\frac{2}{s^3}}{s(s^2-3)} + \frac{s}{s(s^2-3)} + \frac{3}{s(s^2-3)}$$

$$Y(s) = \frac{2}{s^4(s^2-3)} + \frac{1}{s^2-3} + \frac{3}{s(s^2-3)}$$

$$Y(s) = \frac{2 + s^4 + 3s^3}{s^4(s^2-3)}$$

RASTAV NA PARCIJALNE ~~RAZLOM~~ RAZLOMKE

NIJE DOVEDENO DO KRAJA $\emptyset$

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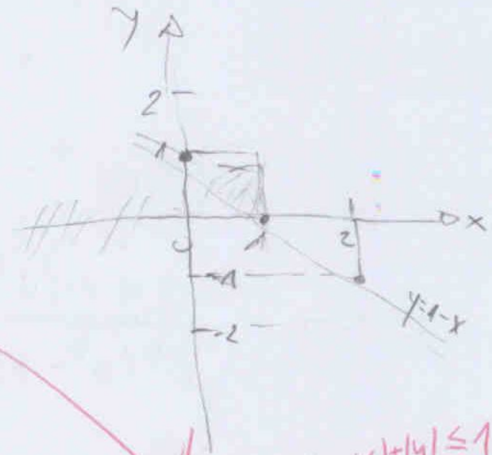
4.) $|x|+|y| \leq 1$ $\qquad \iint\limits_{(S)} xy\,dx\,dy$



$x+y=1$

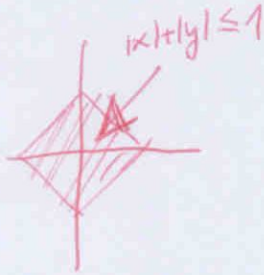
$y=1-x$

x	0	1	2
y	1	0	-1

INTEGRIRANO JE KRIVO PODRUČJE VIDI SKICU

$x+y \leq 1$

$x \leq 1-y \qquad y \leq 1-x$



$$\int_0^1 dx \int_{1-x}^1 xy\,dy = \int_0^1 x\,dx \cdot \frac{y^2}{2}\Big|_{1-x}^{1} =$$

$$= \int_0^1 x \cdot \left(\frac{1}{2} - \frac{(1-x)^2}{2}\right) dx$$

$$= \int_0^1 \frac{1}{2}x\,dx - \int_0^1 x \cdot \left(\frac{1-2x+x^2}{2}\right) dx$$

$$= \frac{1}{2} \int_0^1 x \cdot dx - \int_0^1 \left(\frac{x}{2} - \frac{2x^2}{2} + \frac{x^3}{2}\right) dx$$

$$= \frac{1}{2}\,\frac{x^2}{2}\Big|_0^1 - \frac{1}{2}\int_0^1 x\,dx + \int_0^1 x^2 dx - \frac{1}{2}\int_0^1 x^3 dx$$

$$= \frac{1}{2} \cdot \left(\frac{1}{2}\right) - \frac{1}{2} \cdot \frac{x^2}{2}\Big|_0^1 + \frac{x^3}{3}\Big|_0^1 - \frac{1}{2} \cdot \frac{x^4}{4}\Big|_0^1$$

$$= \frac{1}{4} - \frac{1}{2} \cdot \left(\frac{1}{2}\right) + \frac{1}{3} - \frac{1}{2} \cdot \left(\frac{1}{4}\right)$$

$$= \cancel{\frac{1}{4}} - \cancel{\frac{1}{4}} + \frac{1}{3} - \frac{1}{8} = \frac{1}{3} - \frac{1}{8} = \frac{8-3}{24} = 5/24$$