OBAVEZNO POPUNITI VRIJEME RJEŠAVANJA ISPITA: OD DO

MATEMATIKA 2: Trajanje 120 minuta. Zabranjen je razgovor sa drugim studentima. Na klupama je dozvoljen samo pisaći pribor, tablica osnovnih integrala, kalkulator, indeks ili iksica i prazni papiri koji nose ime studenta. Sav ostali pribor, formule, uređaji, bilješke i nepotpisane prazne papire zabranjeno je koristiti i trebaju ostati u torbi ili pohranjeni kod nastavnika (elektronički uređaji trebaju biti isključeni) tokom cijelog trajanja ispita. Studenti koji primijete zabranjene predmete dužni su ih prijaviti nastavniku. Nije dozvoljeno međusobno posuđivanje pribora tijekom trajanja ispita. Povreda ovih pravila može za posljedicu imati udaljavanje s ispita.

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ZADATKE RIJEŠAVATE JEDNOSTRANO NA PAPIRE KOJE DOBIJETE OD NASTAVNIKA.

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1. Riješiti integrale:
   (a) $\int 2x^2 e^x \, dx$, 8
   (b) $\int \frac{2x+1}{x^2+x-1} \, dx$. 5
2. Integriranjem odrediti površinu trokuta koji je zadan točkama $A(1,3)$, $B(0,0)$, $C(3,1)$. 15
3. Odrediti ekstreme funkcije: $f(x,y) = 3x - 4y - x^2 + xy - y^2$.
4. Riješiti diferencijalnu jednadžbu: $y' + y = 2e^x$.
5. Razviti funkciju $f(x) = \sin(2x)$ u Taylorov red oko točke $x_0 = \frac{\pi}{2}$. Izračunati i izraziti aproksimaciju Taylorovim razvojem sa članovima koji uključuju barem sve potencije manje od x^4. 10

1. a) $\int 2x^2 e^x \, dx = 2\int x^2 e^x \, dx = \left| \begin{matrix} u = x^2 & du = 2x \\ dv = e^x & v = \int e^x dx \\ & v = e^x \end{matrix} \right| =$

$= u \cdot v - \int v \, du$

$= (x^2 \cdot e^x - \int e^x \cdot 2x \, dx) \cdot 2$

$= (x^2 e^x - \int e^x \cdot 2x \, dx) \cdot 2$

$= 2(x^2 e^x + 2xe^x - 2e^x)$ ✓

$= 2x^2 e^x + 4xe^x - 2e^x$

$= 2e^x(x^2 + 2x - 1) + C$

$\int e^x \cdot 2x \, dx = \left| \begin{matrix} u = 2x & du = 2 \\ dv = e^x & v = \int e^x dx \\ & v = e^x \end{matrix} \right|$

$= 2x \cdot e^x - \int e^x \cdot 2 \, dx$

$= 2x \cdot e^x - 2\int e^x dx$

$= 2x \cdot e^x - 2e^x$

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b) $\int \frac{2x+1}{x^2+x-1} \, dx =$

$= \int \frac{2x+1}{(x + \frac{1+\sqrt{5}}{2})(x + \frac{1-\sqrt{5}}{2})} \, dx$

$2x+1 = \frac{A}{(x + \frac{1+\sqrt{5}}{2})} + \frac{B}{(x + \frac{1-\sqrt{5}}{2})}$

$= A(x - 0{,}61) + B(x + 1{,}61)$

$= Ax - 0{,}61A + Bx + 1{,}61B$

$A + B = 2 \Rightarrow A = 2 - B = 2 - \frac{3}{2} = \frac{1}{2}$

$-A + B = 1$ $\quad A = \frac{1}{2}$ ✗

$2B = 3$

$B = \frac{3}{2}$ ✗

$x^2 + x - 1 = 0$

$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x_{1,2} = \frac{-1 \pm \sqrt{1+4}}{2} = \frac{-1 \pm \sqrt{5}}{2}$

$x_1 = \frac{-1+\sqrt{5}}{2} = 0{,}61 \quad x_2 = \frac{-1-\sqrt{5}}{2} = -1{,}61$

$a(x - x_1)(x - x_2)$

$(x + \frac{1+\sqrt{5}}{2})(x + \frac{1-\sqrt{5}}{2})$

$(x - 0{,}61)(x + 1{,}61)$

$x^2 + 1{,}61x - 0{,}61x - 0{,}98$

$x^2 + x - 0{,}98$

Nastavak ispod 5. zadatka

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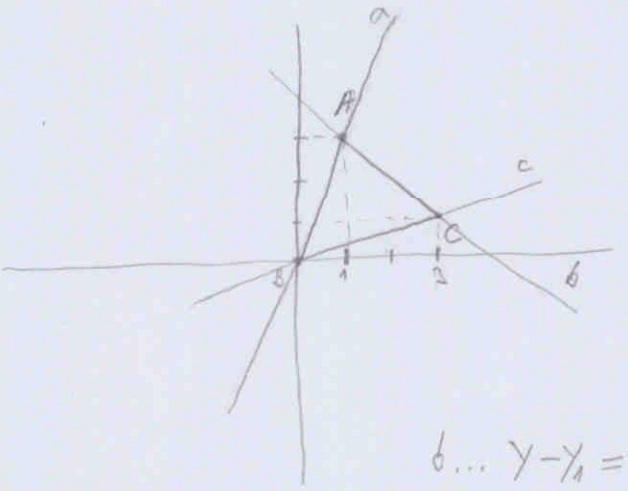
2. $A(1,3)$
$B(0,0)$
$C(3,1)$



a... $y-y_1=\frac{y_2-y_1}{x_2-x_1}(x-x_1)$

$y-3=\frac{0-3}{0-1}(x-1)$

$=3$

$y-3=-3x+3$ $3x-3$

$y=-3x+6$ ✗ $y=3x$

b... $y-y_1=\frac{y_2-y_1}{x_2-x_1}(x-x_1)$

$y-3=\frac{1-3}{3-1}(x-1)$

$y-3=\frac{-2}{2}(x-1)$

$y-3=-1x+1$

$y=-x+4$ ✓

$-3x+6=-x+4$

$-3x+x=4-6$

$-2x=-2$

$x=1$ ✓

c... $y-0=\frac{1-0}{3-0}(x-0)$

$y=\frac{1}{3}x$ ✓

$$P=\int_0^1 -3x+6\,dx+\int_1^3 -x+4\,dx-\int_0^3 \frac{1}{3}x\,dx=$$ ✓ ✓ ✓

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$$=-3\int_0^1 x\,dx+6\int_0^1 dx+\int_1^3 -x+\int_1^3 4\,dx-\frac{1}{3}\int_0^3 x\,dx=$$

$$=-3\frac{x^2}{2}\Big|_0^1+6x\Big|_0^1-\frac{x^2}{2}\Big|_1^3+4x\Big|_1^3-\frac{1}{3}\frac{x^2}{2}\Big|_0^3=$$

$$=\left(-3\frac{1}{2}+6\right)-\left(\frac{6}{2}-\frac{1}{2}\right)+(12-4)-\left(\frac{9}{6}\right)=\frac{-3+12}{2}-\frac{5}{2}+\frac{16}{2}-\frac{9}{6}=$$

$$=\frac{9}{2}-\frac{5}{2}+\frac{16}{2}-\frac{9}{6}=\frac{9-5+16}{2}-\frac{9}{6}=\frac{20}{2}-\frac{9}{6}=10-\frac{9}{6}=\frac{60-9}{6}=\frac{51}{6}$$

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5. $f(x) = \sin(2x)$ $\quad x_0 = \frac{\pi}{2}$

$$f(x) = f(x_0) + \frac{f'(x_0)}{1!}(x-x_0) + \frac{f''(x_0)}{2!}(x-x_0)^2 + \frac{f'''(x_0)}{3!}(x-x_0)^3$$

$f'(x) = 2\cdot\cos(2x)$ ✓
$f''(x) = -4\sin(2x)$
$f'''(x) = -8\cos(2x)$

$$\sin(2x) = 0 + \frac{-2}{1}\left(x-\frac{\pi}{2}\right) + 0 + \frac{8}{6}\left(x-\frac{\pi}{2}\right)^3 \quad \times$$

$$\sin(2x) = -2\left(x-\frac{\pi}{2}\right) + \frac{4}{3}\left(x-\frac{\pi}{2}\right)^3$$

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$f(x_0) = \sin(2x) = 0$
$f'(x_0) = -2$ ✓
$f''(x_0) = 0$
$f'''(x_0) = 8$

1.6) Nastavak

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$$\int \frac{\frac{1}{2}}{\left(x+\frac{1+\sqrt{5}}{2}\right)} + \frac{\frac{3}{2}}{\left(x+\frac{1-\sqrt{5}}{2}\right)}\,dx$$

$$= \frac{1}{2}\int \frac{dx}{\left(x+\frac{1+\sqrt{5}}{2}\right)} + \frac{3}{2}\int \frac{dx}{\left(x+\frac{1-\sqrt{5}}{2}\right)} = \underset{\times}{\frac{1}{2}}\ln\left|x+\frac{1+\sqrt{5}}{2}\right| + \underset{\times}{\frac{3}{2}}\ln\left|x+\frac{1-\sqrt{5}}{2}\right| + C \quad \checkmark$$

$$\frac{2x+1}{-//-} = \frac{A}{\left(x+\frac{1+\sqrt{5}}{2}\right)} + \frac{B}{\left(x+\frac{1-\sqrt{5}}{2}\right)} = \ldots = \frac{1}{x+\frac{1+\sqrt{5}}{2}} + \frac{1}{x+\frac{1-\sqrt{5}}{2}}$$

$$2x+1 = Bx + B\frac{1+\sqrt{5}}{2} + Ax + A\frac{1-\sqrt{5}}{2}$$

$$= x(A+B) + \frac{B+B\sqrt{5}+A-A\sqrt{5}}{2}$$

$$2 = A+B$$

$$1 = \frac{A+B}{2} + \frac{\sqrt{5}}{2}(B-A) \quad \Rightarrow A=1, B=1$$

$$1 = \frac{2}{2} + \frac{\sqrt{5}}{2}(B-A) \Rightarrow B-A=0 \Rightarrow B=A$$