MATEMATIKA 2: Trajanje 120 minuta. Zabranjen je razgovor sa drugim studentima. Na klupama je dozvoljen samo pisaći pribor, tablica osnovnih integrala, kalkulator, indeks ili iksica i prazni papiri koji nose ime studenta. Sav ostali pribor, formule, uređaji, bilješke i nepotpisane prazne papire zabranjeno je koristiti i trebaju ostati u torbi ili pohranjeni kod nastavnika (elektronički uređaji trebaju biti isključeni) tokom cijelog trajanja ispita. Studenti koji primijete zabranjene predmete dužni su ih prijaviti nastavniku. Nije dozvoljeno međusobno posuđivanje pribora tijekom trajanja ispita. Povreda ovih pravila može za posljedicu imati udaljavanje s ispita.

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ZADATKE RIJEŠAVATE NA OVAJ PAPIR.

Broj ↓ bodova

IME I PREZIME: VICE VUŠIĆ **BROJ INDEKSA:** 57102

1. Riješiti integrale:
   (a) $\int \frac{x^3}{x^2+3x+5}\,dx$, ∅
   (b) $\int \frac{\ln x}{x^2}\,dx$. ∅
2. Odrediti površinu između krivulja $y = x+2$ i $y = 4+x-x^2$. 20
3. Odrediti ekstreme funkcije $f(x,y) = 3x^2 + xy - y^3 + 2$. 20
4. Riješiti diferencijalnu jednadžbu: $y'' + y' - 2y = 1$.
5. Razviti funkciju $f(x) = \cos x$ u Taylorov red po potencijama $x - \frac{\pi}{2}$. Izračunati barem prva 4 člana. 10

2. $y = x+2$ $\quad y = 4+x-x^2$

$x+2 = 4+x-x^2$

$x+2-4-x+x^2 = 0$

$x^2+0x-2 = 0$

$x = \frac{0 \pm \sqrt{0+8}}{2}$

$x_1 = \frac{\sqrt{8}}{2}$ $\quad x_2 = -\frac{\sqrt{8}}{2}$

$x_1 = 1{,}41$ ✓ $\quad x_2 = -1{,}41$ ✓

$x+2 = 0$

$4+x-x^2 = 0$

$y = x+2$ $\quad y = x+2$

$y = 1{,}41+2$ $\quad y = -1{,}41+2$

$y_1 = 3{,}41$ $\quad y_2 = 0{,}59$

$S_1(1{,}41;\ 3{,}41)$ $\quad S_2(-1{,}41;\ 0{,}59)$

NULTOČKE

$y = x+2$

$x+2 = 0$

$x = -2$

$4+x-x^2$

$a = -1 < 0 \Rightarrow \cap$

$x = \frac{-1 \pm \sqrt{1+16}}{-2}$

$x = \frac{-1 \pm \sqrt{17}}{-2}$

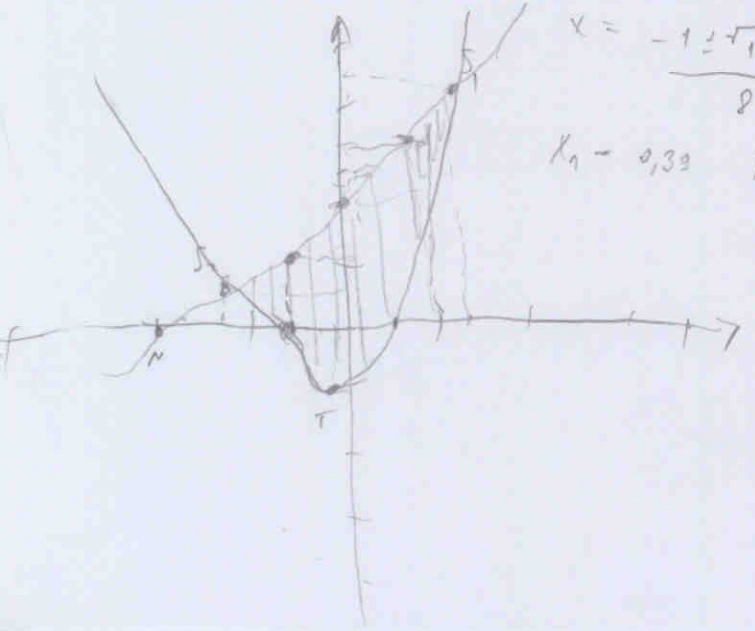
$x_1 = 0{,}39$ $\quad x_2 = -0{,}64$

$T\left(-\frac{b}{2a}, \frac{4ac-b^2}{4a}\right)$

$T\left(-\frac{1}{-2}, \frac{-16-1}{-4}\right)$

$T(-0{,}125;\ -1{,}06)$

x	0	-1	1
$y = x+2$	2	1	3



1.

IME I PREZIME: VICE VIŠE! BROJ INDEKSA: 57/02

$$P = \int_{-1,71}^{1,71} \left[x+2-\left(4+x-x^2\right) \right] dx$$

$$P = \int_{-1,71}^{1,71} \left[x+2-4-x+x^2 \right] dx$$

$$P = \int_{-1,71}^{1,71} \left(x^2-2 \right) dx$$

$$P = \left(\frac{x^3}{3} - 2x \right) \Big|_{-1,71}^{1,71} \quad \checkmark \qquad 20$$

$$P = \frac{1,71^3}{3} - 2\cdot 1,71 - \left(\frac{-1,71^3}{3} - 2\cdot(-1,71) \right)$$

$$P = -1,9 - \left(-0,93 + 2,82 \right)$$

$$P = -1,9 + 0,93 - 2,82 = 3,79 \quad \checkmark$$

3. $f(x,y) = 3x^2+xy-y^2+2$

$\frac{\partial z}{\partial x} = 6x+1$ ✓ $\quad \frac{\partial z}{\partial y} = 1-2y$ ✓ $\quad \frac{\partial^2 z}{\partial x \partial y} = 0$ ✓

$\frac{\partial^2 z}{\partial x^2} = 6$ ✓ $\quad \frac{\partial^2 z}{\partial y^2} = -2$ ✓ $\quad \frac{\partial^2 z}{\partial y \partial x} = 0$ ✓

$D_1 = 6$ $\qquad D_2 = \begin{vmatrix} \frac{\partial^2 z}{\partial x^2} & \frac{\partial^2 z}{\partial y \partial x} \\ \frac{\partial^2 z}{\partial x \partial y} & \frac{\partial^2 z}{\partial y^2} \end{vmatrix} = \begin{vmatrix} 6 & 0 \\ 0 & -2 \end{vmatrix}$ 20

$D_2 = -12$ $\qquad D_2 = -12 < 0$ pa u točki T nema ekstrema ✓

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5. $f(x) = f(x_0) + (x-x_0)\cdot f'(x_0) + \frac{(x-x_0)^2}{2!}\cdot f''(x_0) + \frac{(x-x_0)^3}{3!}\cdot f'''(x_0) + \dots$

$f(x) = \cos x$ ✓ $\Rightarrow f(x_0) = \cos\frac{\pi}{2} = 1$ ✗

$f'(x) = -\sin x$ ✓

$f'(x_0) = -\sin\frac{\pi}{2}$

$f''(x) = -\cos x$ ✓

$f''(x_0) = -\cos\frac{\pi}{2}$

$f'''(x) = \sin x$ ✓

$f'''(x_0) = \sin\frac{\pi}{2}$

$f^{IV}(x) = \cos x$ ✓

$f^{IV}(x_0) = \cos\frac{\pi}{2}$

10

$R(x) = f(\frac{\pi}{2}) + (x-\frac{\pi}{2})\cdot(-\sin\frac{\pi}{2}) + \frac{(x-\frac{\pi}{2})^2}{2!}\cdot(-\cos\frac{\pi}{2}) + \frac{(x-\frac{\pi}{2})^3}{3!}\cdot\sin x + \frac{(x-\frac{\pi}{2})^4}{4!}\cdot\cos x + \frac{(x-\frac{\pi}{2})^5}{5!}\dots$

= TREBALO JE UVRSTITI KONKRETNE BROJEVE

IME I PREZIME: VICE VIŠIĆ BROJ INDEKSA: I7/02

1. a) $\int \frac{x^3}{x^2+3x+5}\,dx$

$u = x^3 \quad dv = x^2+3x+5$

$du = 3x\,dx \quad v = \int x^2+3x+5$

$v = \frac{x^3}{3} + 3\cdot\frac{x^2}{2}$

$v = \frac{x^3}{3} + \frac{3x^2}{2}$

$= x^3 \cdot \left(\frac{x^3}{3} + \frac{3x^2}{2}\right) - \int \left(\frac{x^3}{3} + \frac{3x^2}{2}\right) \cdot 3x\,dx$

$= \frac{x^6}{3} + \frac{3x^2}{2} - \frac{x^5}{5} - 3\cdot\frac{x^3}{3}\cdot 3x\,dx$

$= \frac{x^1}{3} + \frac{3x^2}{2} - \frac{x^5}{5} - \frac{3x^3}{3}\cdot 3x\,dx$

b) $\int \frac{\ln x}{x^2}\,dx$

$u = \ln x \quad dv = x^{-2}dx$

$du = \frac{1}{x}\,dx \quad v = \int x^{-2}dx$

$v = \frac{x^{-1}}{-1}$

$dv = \frac{1}{x^2}dx$

$\ln x \cdot \frac{x^{-1}}{-1} - \int \frac{x^{-1}}{-1}\cdot\frac{1}{x}\,dx$

$u = \frac{1}{x} \quad dv = \frac{x^{-1}}{-1}dx$

$du = \frac{1}{x^2}\,dx \quad v = \int \frac{x^{-1}}{-1}$

$v = \frac{x^0}{0}$

$= \frac{1}{x}\cdot\frac{x^0}{0} - \int \frac{x^0}{0}\cdot\frac{1}{x^2}\,dx$