MATEMATIKA 1: Ispit traje 100 minuta. Zabranjen je razgovor sa drugim studentima. Na klupama je dozvoljen samo pisaći pribor, kalkulator i indeks ili iksica i prazni papiri koji nose ime studenta. Sav ostali pribor, formule, uređaji, bilješke i nepotpisane prazne papire zabranjeno je koristiti i trebaju ostati u torbi ili pohranjeni kod nastavnika (elektronički uređaji trebaju biti isključeni) tokom cijelog trajanja ispita. Studenti koji primijete zabranjene predmete dužni su ih prijaviti nastavniku. Nije dozvoljeno međusobno posuđivanje pribora tijekom trajanja ispita. Povreda ovih pravila može za posljedicu imati udaljavanje s ispita. ZADATKE RIJEŠAVATE NA OVAJ PAPIR. oxox

Broj ↓ bodova

IME I PREZIME: MAJA ŠOGORIĆ **BROJ INDEKSA:** 56078

1. Riješiti jednadžbu za kompleksni broj z: $\overline{2+2i} = \dfrac{z^3}{z^3+i^{21}}$. 20

$z_3 = 8^1\left(\sin \frac{7\pi}{4} + 2\cdot 2\cdot\pi + \cos \frac{7\pi}{4} + 4\pi\right)$

$z_3 = 8^1\left(\sin \frac{23\pi}{4} + \cos \frac{23\pi}{4}\right)$

2. Da li postoji inverz matrice A i za¨to? Ako postoji A^{-1}.

$$A = \begin{bmatrix} 0 & 2 & 0 & 1 \\ 2 & 0 & 1 & 0 \\ 0 & 1 & 0 & 2 \\ 1 & 0 & 2 & 0 \end{bmatrix}$$

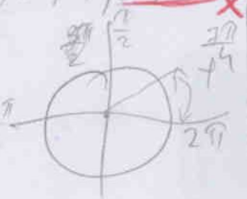
3. Ispitati tok funkcije: $f(x) = \dfrac{\sqrt{x^2-1}}{x}$. 40

$\sqrt[3]{\sqrt{8}} = \sqrt[3]{8^{\frac{1}{2}}}$

$= 8^{\frac{1}{2}+\frac{1}{3}} = 8^{\frac{3+2}{6}=\frac{6}{6}} = 8^1$

4. Pronaći sve ekstreme funkcije: $g(x) = \dfrac{\ln 2x}{\sqrt{x}}$. 20

$2+2i = \dfrac{z^3}{z^3+i^{21}} \Big/ (i^2) = -1,\ (i^4) = 1,\ (i^{21}) = 1$ ✗



$2+2i = \dfrac{z^3}{z^3+1} \Big/ \cdot (z^3+1)$

$(2+2i)(z^3+1) = z^3$

$x\ y$
$2+2i$

$2z^3 + 2 + 2z^3 i + 2i = z^3$

$r = |x+y| = \sqrt{2^2+2^2} = \sqrt{4+4} = \sqrt{8}$

$\cancel{2z^3} + 2 - \cancel{2z^3} + 2 = z^3$ ✗

$4 = z^3$ ✗

$\operatorname{tg} \varphi = \frac{y}{x} = \frac{2}{2} = 1,\ \varphi = 45^0$

$z^n = \sqrt[n]{r}\left[\sin \frac{\varphi+2k\pi}{\pi} + i\cos \frac{\varphi+2k\pi}{\pi}\right)$

$z^3 = \sqrt[3]{\sqrt{8}}\left(\sin \frac{\frac{7\pi}{4}+2k\pi}{} + i\cos \frac{\frac{7\pi}{4}+2k\pi}{}\right.$

$z_1 = 8^1\left(\sin \frac{7\pi}{4} + 2\cdot 0\cdot\pi + \cos \frac{7\pi}{4}\right)$

$z_2 = 8^1\cdot\left(\sin \frac{7\pi}{4} + 2\cdot 1\cdot\pi + \cos \frac{7\pi}{4} + 2\pi\right)$

$z_2 = 8\left(\sin \frac{15\pi}{4} + \cos \frac{15\pi}{4}\right)$

✗ Ø

$f'(x^2-1)^{\frac{1}{2}}$ ✗

$\frac{1}{2}(x^2-1)\cdot(x^2)' \cdot 1 + x^2\cdot(-1)'$

$\left(\frac{1}{2}x^2 - \frac{1}{2}\right)\cdot 2x\cdot 1 + x^2\cdot 0$

$\frac{2}{2}x^3 - \frac{2}{2}x = x^3 - x$

(-1)

$2 - \frac{1}{2} = \frac{}{2}$

3. $f(x) = \dfrac{\sqrt{x^2-1}}{x}$

1) $D = 2$ ✗

2) $f(-1) = \dfrac{\sqrt{(-1)^2-1}}{-1} = \dfrac{\sqrt{1-1}}{-1} = \dfrac{0}{-1} = -\infty$

$f(1) = \dfrac{\sqrt{1^2-1}}{1} = \dfrac{\sqrt{1-1}}{1} = \dfrac{0}{1} = +\infty$

3) −

4) $x^2 - 1 = 0$ $\quad x_2 = 0$

$x_1 = 1$

5) $l(x) = \lim \dfrac{\sqrt{x^2-1}}{x}$ $\quad 0$

$f(x) = \lim\limits_{x\to-\infty} \dfrac{(x^2-1)^{\frac{1}{2}}}{x} = \lim\limits_{x\to-\infty} \dfrac{\left(\frac{x^2}{x} - \frac{1}{x}\right)^{\frac{1}{2}}}{\frac{x}{x}}$?

(2) $\lim\limits_{x\to-\infty} \dfrac{x^{\frac{1}{2}}}{1} = \lim\limits_{x\to-\infty} \sqrt{x}$?

$f(x) - l(x) = \lim\limits_{x\to+\infty} \dfrac{\sqrt{x^2-1}}{x} - \sqrt{x} \Big/ \cdot x = \lim\limits_{x\to\infty} \sqrt{x^2-1} - \sqrt{x}$

$\lim\limits_{x\to+\infty} (x^2-1)^{\frac{1}{2}} - (x)^{\frac{1}{2}} = \lim\limits_{x\to+\infty} x^{\frac{3}{2}} - 1 - x^{\frac{1}{2}} = \lim\limits_{x\to\infty} x^2 - 1$

6) $f'(x) = \dfrac{\sqrt{x^2-1}}{x} = \dfrac{(\sqrt{x^2-1})'\cdot x - (\sqrt{x^2-1})\cdot x'}{x^2}$ ✓

✗

$f'(x) = \dfrac{x^3 - x - (x^2-1)^{\frac{1}{2}}}{x^2}$ ✗ $= \dfrac{x^3 - x - x^{\frac{3}{2}} + 1^{\frac{1}{2}}}{x^2}$

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IME I PREZIME: MAJA ŠOGORIĆ BROJ INDEKSA: 38078

$(\sqrt{x}-1)' =$
$(\sqrt{x})' \cdot 1 - \sqrt{x} \cdot (1)'$
$\frac{1}{2} x \cdot 1 - \sqrt{x} \cdot 0$
$\frac{1}{2} x$

3./NASTAVAK

$$f'(x) = \frac{x^3 - x' - x^{\frac{3}{2}} - 1^{\frac{1}{2}}}{x^2} = \frac{x^2 - x^{\frac{3}{2}} - \sqrt{1}}{x^2} = \frac{x^{\frac{1}{2}} - \sqrt{1}}{x^2} = \frac{\sqrt{x} - \sqrt{1}}{x^2} = \frac{\sqrt{x}-1}{x^2}$$

$$f''(x) = \left(\frac{\sqrt{x} - \sqrt{1}}{x^2}\right)' = \frac{(\sqrt{x} - \sqrt{1})' \cdot x^2 - (\sqrt{x} - \sqrt{1}) \cdot (x^2)'}{x^4} = \frac{\frac{1}{2} x \cdot x^2 - (\sqrt{x}-1) \cdot 2x}{x^4}$$

$$f''(x) = \frac{\frac{1}{2} x^3 - (\sqrt{x}-1) \cdot 2x}{x^4} = \frac{\frac{1}{2} x^3 - (2x \cdot x^{\frac{1}{2}} - 2x)}{x^4} = \frac{\frac{1}{2} x^3 - 2x^{\frac{3}{2}} + 2x}{x^4}$$

$$f''(x) = \frac{\frac{1}{2} x^3 - 2\sqrt[2]{x^3} + 2x}{x^4}$$

$$f(x) = \frac{\sqrt{x^2-1}}{x}$$

$$f'(x) = \frac{(\sqrt{x^2-1})' \cdot x - \sqrt{x^2-1} \cdot x'}{x^2} = \frac{\frac{1}{\cancel{2}} \frac{\cancel{2}x}{\sqrt{x^2-1}} \cdot x - \sqrt{x^2-1} \cdot 1}{x^2}$$

$$= \frac{\frac{x^2}{\sqrt{x^2-1}} - \frac{x^2-1}{\sqrt{x^2-1}}}{x^2} = \frac{1}{x^2\sqrt{x^2-1}}$$