

MATEMATIKA 3: Trajanje 120 minuta. Zabranjen je razgovor sa drugim studentima. Na klupama je dozvoljen samo pišaći pribor, tablica osnovnih integrala, tablica Laplaceovih transformacija, kalkulator, indeks ili iksica i prazni papiri koji nose ime studenta. Sav ostali pribor, formule, uređaji, bilješke i nepotpisane prazne papire zabranjeno je koristiti i trebaju ostati u torbi ili pohranjeni kod nastavnika (elektronički uređaji trebaju biti isključeni) tokom cijelog trajanja ispita. Studenti koji primijete zabranjene predmete dužni su ih prijaviti nastavniku. Nije dozvoljeno međusobno posuđivanje pribora tijekom trajanja ispita. Povreda ovih pravila može za posljedicu imati udaljavanje s ispita. ZADATKE RIJEŠAVATE JEDNOSTRANO NA PAPIRE KOJE DOBIJETE OD NASTAVNIKA.

M3

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IME I PREZIME: VINKO RADOVIĆ

BROJ INDEKSA: 54678

1. Izračunati dvostruki integral:

$$\iint_S xy \, dx \, dy,$$

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gdje je S područje omeđeno kružnicom $x^2 + y^2 = 4$ i pravcem koji prolazi točkama $A(0, 2)$ i $B(2, 0)$.

2. Izračunati površinu lika omeđenog pravcem
- $2x + y + 1 = 0$
- i parabolom
- $y^2 = 1 + x$
- .

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3. Izračunati volumen tijela omeđenog valjkom
- $x^2 + 4y^2 = 4$
- i ravninama
- $z = y$
- i
- $z = -2$
- .

4. Izračunati

$$\int_{(0, \pi)}^{(2, 2\pi)} 2x \sin y \, dx + (x^2 + 1) \cos y \, dy$$

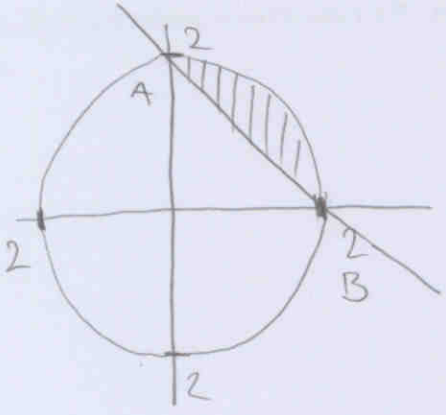
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5. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu:

$$2x'''(t) + 5x'(t) = t, \quad x'(0) = x''(0) = 0, \quad x(0) = 1.$$

v. 25

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$$x^2 + y^2 = 4 \quad y = \sqrt{4 - x^2}$$

P... A(0,2) ; B(2,0)

$$y - y_A = \frac{y_B - y_A}{x_B - x_A} (x - x_A)$$

$$y - 2 = \frac{-2}{2} (x - 0)$$

$$y = -x + 2$$

$$\iint_S xy \, dx \, dy = \int_0^2 \int_{-x+2}^{\sqrt{4-x^2}} xy \, dy \, dx \quad \checkmark$$

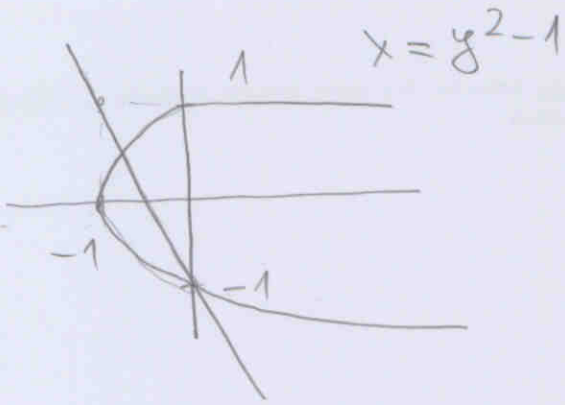
$$= \int_0^2 \left(\frac{xy^2}{2} \right) \Big|_{y=-x+2}^{\sqrt{4-x^2}} dx = \int_0^2 \left(\frac{x(4-x^2)}{2} - \frac{x(2-x)^2}{2} \right) dx \quad \checkmark$$

$$= \int_0^2 \frac{-x^3 + 4x - x^3 + 4x^2 - 4x}{2} dx$$

$$= \int_0^2 (-x^3 + 2x^2) dx = \left(-\frac{x^4}{4} + \frac{2x^3}{3} \right) \Big|_{x=0}^2$$

$$= -\frac{16}{4} + \frac{16}{3} = \frac{-12 + 16}{3} = \frac{4}{3} \quad \checkmark$$

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$$x = x$$

$$y^2 - 1 = \frac{-y - 1}{2} \quad | \cdot 2$$

$$2y^2 + y - 1 = 0$$

$$y_{1,2} = \frac{-1 \pm 3}{4}$$

$$y_1 = -1 \quad y_2 = \frac{1}{2} \quad \checkmark$$

$$P = \int_{-1}^{\frac{1}{2}} \int_{y^2-1}^{\frac{-y-1}{2}} dx dy = \int_{-1}^{\frac{1}{2}} \left(-\frac{1}{2}(y+1) - y^2 + 1 \right) dy \quad \checkmark$$

$$P = \left(-\frac{1}{2} \left(\frac{y^2}{2} + y \right) - \frac{y^3}{3} + y \right) \Big|_{y=-1}^{\frac{1}{2}}$$

$$= -\frac{1}{2} \cdot \left(\frac{1}{8} + \frac{1}{2} \right) - \frac{1}{24} + \frac{1}{2} - \left(-\frac{1}{2} \left(\frac{1}{2} - 1 \right) + \frac{1}{3} - 1 \right)$$

$$P = -\frac{5}{16} - \frac{1}{24} + \frac{1}{2} - \left(\frac{1}{4} - \frac{1}{3} - 1 \right)$$

$$= \frac{-15 - 2 + 24 - 12 - 16 + 48}{48} = \frac{17}{48} \quad \checkmark$$

50) $2x'''(t) + 5x'(t) = 1 \quad x'(0) = x''(0) = 0 \quad x(0) = 1$

$s^3 F(s) - s^2 f(0) - s f'(0) - f''(0)$

$s^3 F(s) - s^2$

$s F(s) - f(0)$

$s F(s) - 1$

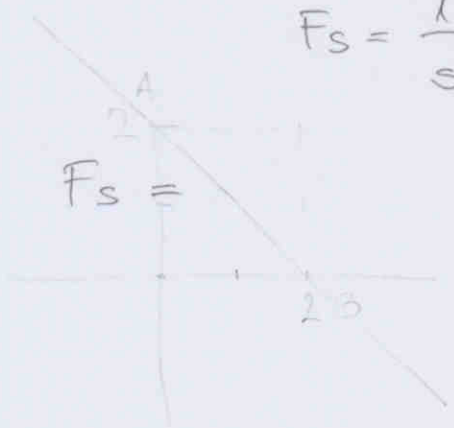
$2(s^3 F(s) - s^2) + 5(s F(s) - 1) = \frac{1}{s^2}$

$2s^3 F(s) + 5s F(s) = \frac{1}{s^2} + 1 + s^2 \quad \times$

$F(s) [2s^3 + 5s] = \frac{1}{s^2} + 1 + s^2 \quad / : 2s^3 + 5s$

$F_s = \frac{\frac{1}{s^2} + 1 + s^2}{2s^3 + 5s} = \frac{1 + s^2}{s(2s^2 + 5)} + \frac{\frac{1}{s^2}}{2s^3 + 5s}$

$F_s = \frac{1 + s^2}{s(2s^2 + 5)} + \frac{2s + 5}{2s^6 + 5s^3}$



VIDI VIDOVI

$$\int_{(0, \pi)}^{(2, 2\pi)} 2x \sin y dx + (x^2 + 1) \cos y dy$$

$$\left(\begin{array}{l} \int 2x \sin y dx = x^2 \sin y \\ \int (x^2 + 1) \cos y dy = (x^2 + 1) \sin y \end{array} \right) \checkmark$$

RJEŠENJE

SADRŽI OBA

RJEŠENJA

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$$(x^2 + 1) \sin y \Big|_{(x, y) = (0, \pi)}^{(2, 2\pi)} \checkmark$$

$$= 5 \cdot 0 - 1 \cdot 0 = 0 \checkmark$$

$$\nabla F = \begin{pmatrix} \frac{\partial F}{\partial x} \\ \frac{\partial F}{\partial y} \end{pmatrix} = \begin{pmatrix} 2x \sin y \\ (x^2 + 1) \cos y \end{pmatrix}$$

$$\Rightarrow F = \int 2x \sin y dx = x^2 \sin y + C(y)$$

UVRSTITI:

$$(x^2 + 1) \cos y = \frac{\partial F}{\partial y} = x^2 \cos y + C'(y)$$

$$\cancel{x^2 \cos y} + \cos y = \cancel{x^2 \cos y} + C'(y)$$

$$\Rightarrow C'(y) = \cos y$$

$$\Rightarrow C(y) = \int \cos y dy = \sin y$$

$$\Rightarrow F = x^2 \sin y + \sin y = (x^2 + 1) \sin y$$

$$I = F(2, 2\pi) - F(0, \pi)$$

$$= (2^2 + 1) \underbrace{\sin(2\pi)}_{=0} - 1 \cdot \underbrace{\sin(\pi)}_{=0}$$

$$= 0$$