OBAVEZNO POPUNITI VRIJEME RJEŠAVANJA ISPITA: OD 16:43 DO

MATEMATIKA 3: Trajanje 120 minuta. Zabranjen je razgovor sa drugim studentima. Na klupama je dozvoljen samo pisaći pribor, tablica osnovnih integrala, tablica Laplaceovih transformacija, kalkulator, indeks ili iksica i prazni papiri koji nose ime studenta. Sav ostali pribor, formule, uređaji, bilješke i nepotpisane prazne papire zabranjeno je koristiti i trebaju ostati u torbi ili pohranjeni kod nastavnika (elektronički uređaji trebaju biti isključeni) tokom cijelog trajanja ispita. Studenti koji primijete zabranjene predmete dužni su ih prijaviti nastavniku. Nije dozvoljeno međusobno posuđivanje pribora tijekom trajanja ispita. Povreda ovih pravila može za posljedicu imati udaljavanje s ispita. ZADATKE RJEŠAVATE JEDNOSTRANO NA PAPIRE KOJE DOBIJETE OD NASTAVNIKA. M3

40

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1. Izračunati dvostruki integral:

$$\iint_S xy\,\mathrm{d}x\mathrm{d}y\,,$$ 20

gdje je S područje omeđeno kružnicom $x^2+y^2=4$ i pravcem koji prolazi točkama $A(0,2)$ i $B(2,0)$.

2. Izračunati površinu lika omeđenog pravcem $2x+y+1=0$ i parabolom $y^2=1+x$. 15

3. Izračunati volumen tijela omeđenog valjkom $x^2+4y^2=4$ i ravninama $z=y$ i $z=-2$.

4. Izračunati

$$\int_{(0,\pi)}^{(2,2\pi)} 2x\sin y\,\mathrm{d}x+\left(x^2+1\right)\cos y\,\mathrm{d}y$$

5. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 5

$$2x'''(t)+5\,x'(t)=t,\quad x'(0)=x''(0)=0,\ x(0)=1.$$

1.) $x^2+y^2=4$ $A(\overset{x_1}{0},\overset{y_1}{2})$ $B(\overset{x_2}{2},\overset{y_2}{0})$ $y-y_1=\frac{y_2-y_1}{x_2-x_1}\cdot(x-x_1)$

$r=2$

$x^2+y^2=4$

$x^2=4-y^2$

$x=\sqrt{4-y^2}$

$y-2=\frac{0-2}{2-0}\cdot(x-0)$

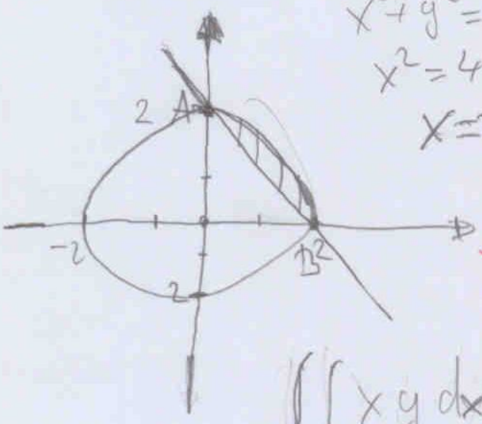
$y-2=\frac{-2}{2}\cdot x$

$y-2=-x$

$y=-x+2$

$-x=y-2\ /\cdot(-1)$

$x=-y+2$



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$$\iint_S xy\,dx\,dy=\int_0^2\int_{-y+2}^{\sqrt{4-y^2}} xy\,dx\,dy=\int_0^2 y\left(\frac{x^2}{2}\right)\Big|_{-y+2}^{\sqrt{4-y^2}}dy$$

$$=\int_0^2 y\cdot\left(\frac{\sqrt{4-y^2}^{\,2}}{2}-\left(\frac{(-y+2)^2}{2}\right)\right)dy=\int_0^2 y\cdot\left(\frac{4-y^2}{2}-\frac{y^2-\overset{4}{2}y+4}{2}\right)dy$$

$$=\int_0^2 y\cdot\left(\frac{4-y^2-(y^2-2y+4)}{2}\right)dy=\int_0^2 y\cdot\left(\frac{4-y^2-y^2+2y-4}{2}\right)dy=\int_0^2 y\cdot\frac{-2y^2+2y}{2}dy$$

$$=\frac{1}{2}\int_0^2 y\cdot(-2y^2+2y)\,dy=\frac{1}{2}\int_0^2 -2y^3+2y^2=\frac{1}{2}\cdot\left(-\frac{2y^4}{4}+\frac{2y^3}{3}\right)\Big|_0^2=\frac{1}{2}\cdot\left(-\frac{2(2)^4}{4}+\frac{2\cdot(2)^3}{3}\right)-$$

$$\left(-\frac{2(0)^4}{4}+\frac{2(0)^3}{3}\right)=\frac{1}{2}\cdot\left(-\frac{2\cdot16}{4}+\frac{2\cdot8}{3}\right)=\frac{1}{2}\cdot\left(-8+\frac{16}{3}\right)=\frac{1}{2}\cdot\left(\frac{-24+16}{3}\right)=\frac{1}{\cancel{2}}\cdot\frac{-\cancel{8}}{3}=-\frac{4}{3}$$

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2) $y^2 = 1 + x$ $\quad 2x + y + 1 = 0 \quad y + 1 = -2x$

$y = -2x - 1$

$2x = -y - 1 \,/ :2$

za $y = 0 \quad 0 = 1 + x$

Folun $x = -1$ $\quad x = -\frac{y}{2} - \frac{1}{2}$

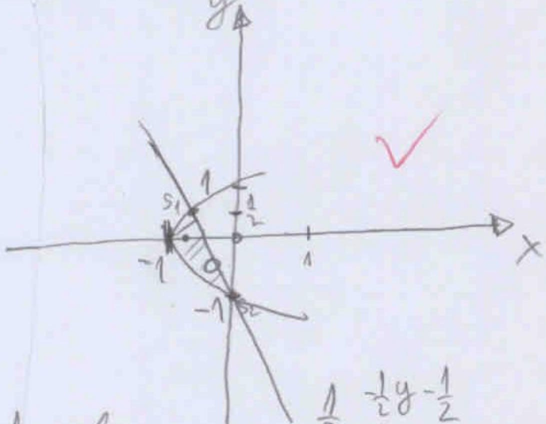
15

$y^2 = 1 + x$

$x = y^2 - 1$

$y^2 = 1 + x$

$x = -\frac{y}{2} - \frac{1}{2} = -\frac{1}{2}y - \frac{1}{2}$



$y^2 = 1 - \frac{y}{2} - \frac{1}{2}$

$y^2 + \frac{y}{2} + \frac{1}{2} - 1 = 0$

$y^2 + \frac{1}{2}y - \frac{1}{2} = 0$

$y_{1,2} = \frac{-\frac{1}{2} \pm \sqrt{(\frac{1}{2})^2 + 4ac}}{2a}$

$P = \iint_S dx\,dy = \int_{-1}^{\frac{1}{2}} \int_{y^2-1}^{-\frac{1}{2}y - \frac{1}{2}} dx\,dy$

$y_{1,2} = \frac{-\frac{1}{2} \pm \sqrt{\frac{9}{4}}}{2} = \frac{-\frac{1}{2} \pm \frac{3}{2}}{2}$

$y_1 = \frac{-\frac{1}{2} + \frac{3}{2}}{2} = \frac{\frac{2}{2}}{2} = \frac{1}{2}$

$= \int_{-1}^{\frac{1}{2}} \left(\frac{x^2}{2}\right)\Big|_{y^2-1}^{-\frac{1}{2}y - \frac{1}{2}} dy = \int_{-1}^{\frac{1}{2}} \left(\frac{(-\frac{1}{2}y - \frac{1}{2})^2}{2} - \left(\frac{(y^2-1)^2}{2}\right) \right) dy$

$y_2 = \frac{-\frac{1}{2} - \frac{3}{2}}{2} = \frac{-\frac{4}{2}}{2} = \frac{-2}{2} = -1$

$x_1 = -\frac{1}{2}y - \frac{1}{2}$

$= -\frac{1}{2} \cdot \frac{1}{2} - \frac{1}{2} = -\frac{1}{4} - \frac{1}{2}$

VIDI RADOVIĆ

$= \frac{-1-2}{4} = \frac{-3}{4}$

$= \int_{-1}^{\frac{1}{2}} \left(\frac{1}{4}y^2 + 2 \cdot \frac{1}{2}y \cdot \frac{1}{2} + \frac{1}{4}\right) - \left(y^4 - 2y^2 + 1\right) dy$

$x_2 = -\frac{1}{2} \cdot (-1) - \frac{1}{2} = 0$

$= \int_{-1}^{\frac{1}{2}} \left(\frac{1}{4}y^2 + \frac{1}{2}y + \frac{1}{4} - y^4 + 2y^2 - 1\right) dy$ $\quad S_1\left(-\frac{3}{4}, \frac{1}{2}\right) \; S_2(0, -1)$

$= \int_{-1}^{\frac{1}{2}} -y^4 + \frac{1}{4}y^2 + 2y^2 + \frac{1}{2}y + \frac{1-4}{4} \,) dy = \int_{-1}^{1/2} -y^4 + \frac{1}{4}y^2 + 2y^2 + \frac{1}{2}y + \frac{-3}{4}$

$= -\frac{y^5}{5} + \frac{1}{4} \cdot \frac{y^3}{3} + 2 \cdot \frac{y^3}{3} + \frac{1}{2} \cdot \frac{y^2}{2} - \frac{3}{4}y \Big|_{-1}^{1/2} = -\frac{(\frac{1}{2})^5}{5} + \frac{1}{4} \cdot \frac{(\frac{1}{2})^3}{3} + 2 \cdot \frac{(\frac{1}{2})^3}{3} + \frac{1}{2} \cdot \frac{(\frac{1}{2})^2}{2}$

$-\frac{3}{4} \cdot \frac{1}{2} - \left(-\frac{(-1)^5}{5} + \frac{1}{4} \cdot \frac{(-1)^3}{3} + 2 \cdot \frac{(-1)^3}{3} + \frac{1}{2} \cdot \frac{(-1)^2}{2} + \frac{3}{4} \cdot \ldots\right.$

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5.) $2x'''(t) + 5x'(t) = t \qquad x'(0) = x''(0) = 0,\ x(0) = 1$

$x'(t) = sX(s) - x(0) = sX(s) - 1$

$x'''(t) = s^3X(s) - s^2\underbrace{x(0)}_{1} - s\underbrace{x'(0)}_{0} - \underbrace{x''(0)}_{0} = s^3X(s) - s^2$

$2\cdot(s^3X(s) - s^2) + 5\cdot(sX(s) - 1) = t$

5

$2s^3X(s) - 2s^2 + 5sX(s) - 5 = \frac{1}{s^2}$

$2s^3X(s) + 5sX(s) = \frac{1}{s^2} + 2s^2 + 5$

$(2\cdot s^3 + 5s)\cdot X(s) = \frac{1}{s^2} + 2s^2 + 5$

$s(2s^2+5)\,X(s) = \frac{1}{s^2} + 2s^2 + 5 \;/ : s(2s^2+5)$ ✓

$X(s) = \dfrac{\frac{1}{s^2} + 2s^2 + 5}{s(2s^2+5)}$ ✓

$\dfrac{\frac{1}{s^2} + 2s^2 + 5}{s(2s^2+5)} = \dfrac{A}{s} + \dfrac{B}{2s^2+5} \;/ \cdot s(2s^2+5)$

$\frac{1}{s^2} + 2s^2 + 5 = A(2s^2+5) + Bs$

$\frac{1}{s^2} + 2s^2 + 5 = 2As^2 + 5A + Bs$

$5A = 5 \qquad B = 0$

$5A = 5$

$A = 1$

$X(s) = \dfrac{\frac{1}{s^2} + 2s^2 + 5}{s(2s^2+5)} = \dfrac{1}{s} + \dfrac{0}{2s^2+5}$

$= \frac{1}{s} + 0$

$\bullet\!-\!\circ \quad X(t) = 1$

X ✓

$X(s) = \dfrac{\frac{1}{s^2} + 2s^2 + 5}{s(2s^2+5)} = \dfrac{1}{s^3(2s^2+5)} + \dfrac{2s^2+5}{s(2s^2+5)} = \dfrac{1 + 2s^4 + 5s^2}{s^3(2s^2+5)} = \dfrac{A}{s} + \dfrac{B}{s^2} + \dfrac{C}{s^3} + \dfrac{Ds+E}{2s^2+5}$

$\left\{A = \frac{23}{25},\ B = 0,\ C = \frac{1}{5},\ D = \frac{4}{25},\ E = 0\right\} \Rightarrow \frac{1}{5s^3} + \frac{23}{25s} + \frac{4s}{25(5+2s^2)} \Rightarrow x(t) = \frac{1}{10}t^2 + \frac{23}{25} + \frac{2}{25}\cos\left(\sqrt{\frac{5}{2}}t\right)$