

MATEMATIKA 3: Trajanje 120 minuta. Zabranjen je razgovor sa drugim studentima. Na klupama je dozvoljen samo pisači pribor, tablica osnovnih integrala, tablica Laplaceovih transformacija, kalkulator, indeks ili iksica i prazni papiri koji nose ime studenta. Sav ostali pribor, formule, uređaji, bilješke i nepotpisane prazne papire zabranjeno je koristiti i trebaju ostati u torbi ili pohranjeni kod nastavnika (elektronički uređaji trebaju biti isključeni) tokom cijelog trajanja ispita. Studenti koji primijete zabranjene predmete dužni su ih prijaviti nastavniku. Nije dozvoljeno međusobno posuđivanje pribora tijekom trajanja ispita. Povreda ovih pravila može za posljedicu imati udaljavanje s ispita. ZADATKE RIJEŠAVATE JEDNOSTRANO NA PAPIRE KOJE DOBIJETE OD NASTAVNIKA.

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IME I PREZIME: FLIP BILAUER

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1. Izračunati dvostruki integral

$$\iint_S e^{x+y} dx dy,$$

gdje je S trokut s vrhovima $A(0,1)$, $B(1,0)$, $C(1,1)$.

2. Izračunati površinu lika omeđenog parabolama $(y+1)^2 = 2x+5$, $y^2 - \frac{y}{2} = -x+3$.

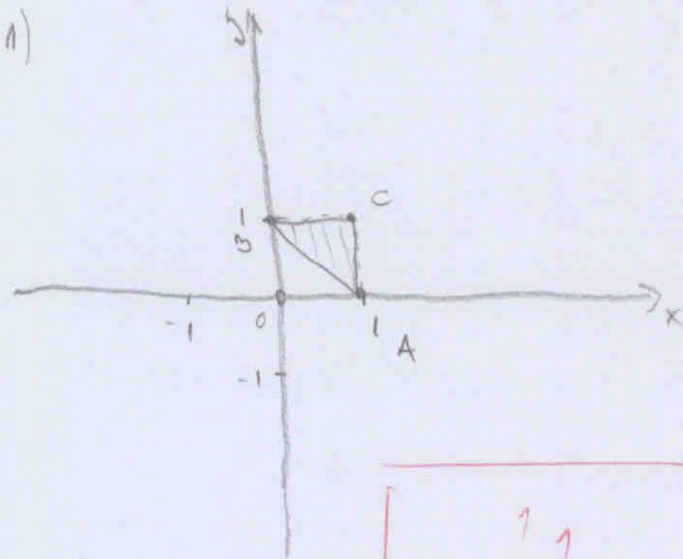
3. Izračunati volumen tijela omeđenog ploham: $z = x^2 + y^2$, $z = 2$.

4. Neka je točkama $A(1,0)$, $B(1,4)$, $C(-2,2)$ $D(-2,0)$ dan četverokut $ABCD$ i neka je C njegova kontura prijeđena u pozitivnom smislu (suprotno od kazaljke na satu). Primjenom Greenove formule izračunati integral

$$\oint_C 2(x+y^2)dx + (x+y)^2 dy$$

5. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu:

$$2f'''(t) + 2f''(t) = 0, \quad f(0) = f''(0) = 2, \quad f'(0) = 0.$$



$$AB: y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} \cdot (x - x_1)$$

$$y - 1 = \frac{0 - 1}{1 - 0} \cdot (x - 0)$$

$$y - 1 = -x$$

$$x = y - 1$$

$$AC: y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} (x - x_1) \quad BC: y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} (x - x_1)$$

$$y - 1 = \frac{1 - 1}{1 - 0} (x - 0)$$

$$y = 1$$

$$y - 0 = \frac{1 - 0}{1 - 1} (x - 1)$$

$$y = 0 \Rightarrow x = 0$$

$$P = \int_0^1 \int_{y-1}^1 e^{x+y} dx dy$$

$$\int_0^1 \int_0^1 e^{x+y} dx dy$$

$$= \int_0^1 \int_0^{y-1} e^x \cdot e^y dx dy$$

$$= \int_0^1 e^y \int_0^{y-1} e^x dx$$

$$= \int_0^1 e^y (e^x)_{x=0}^{x=y-1}$$

$$= \int_0^1 e^y (e^{y-1} - e^0)$$

$$= \int_0^1 e^y \cdot e^{y-1} - e^y$$

$$= \int_0^1 e^{2y-1} - e^y$$

$$= (e^{2y-1})_{y=0}^1 - e^y$$

$$= \int_0^1 e^y (e^{y-1} - e^0) dy$$

$$= \int_0^1 (e^{2y-1} - e^{y+0}) dy$$

$$= \int_0^1 (e^{2y-1} - e^y) dy$$

$$= \left(\frac{e^{2y-1}}{2} - e^y \right)_{y=0}^1$$

$$= \left(\frac{e}{2} - e \right) - \left(\frac{e^{-1}}{2} - e^0 \right)$$

$$= -\frac{e}{2} - \frac{e^{-1}}{2} + 1$$

2. $(y+1)^2 = 2x+5 \Rightarrow y^2 + 2y + 1 = 2x+5$

y	0	1	2
x	2	3	5

$y^2 - \frac{5}{2} = -x + 2$
 $2xy = 0$

$y^2 - \frac{1}{2}y = -x + 3 \quad | \cdot (-1)$

$x = 3 \quad ||$

$y^2 + 2y = 2x + 4 \Rightarrow$

$y^2 + 2y + 1 = \frac{2 \cdot (\frac{5}{2}y - 1) + 4}{3} + 5$

$-y^2 + \frac{1}{2}y = x - 2$

$2xy = 0$

$2x + 4 = 0$

$y^2 + 2y + 1 = \frac{5y - 2}{3} + 5 \quad | \cdot 3$

$\frac{5}{2}y = 3x + 1$

$x = -2$

$3y^2 + 6y + 3 = 5y - 2 + 15$

$3x + 1 = \frac{5}{2}y$

$3y^2 + y - 2 = 0$

$3x = \frac{5}{2}y - 1$ ✓

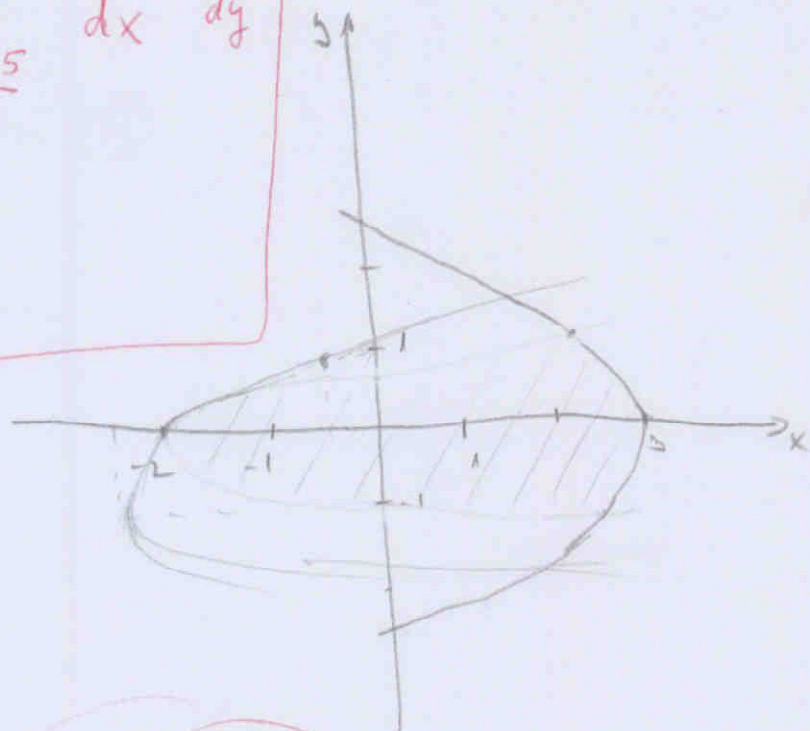
$y_{1,2} = \frac{-1 \pm \sqrt{1 + 24}}{6}$

$x = \frac{1 \pm \frac{5}{2}y - 1}{3}$ ✓

skicista

$y_1 = \frac{4}{6} = \frac{2}{3}$ ✓ $y_2 = -1$ ✓

$P = \int_{-1}^{\frac{2}{3}} \int_{\frac{-1(y+1)^2-5}{2}}^{\frac{2}{3}y^2 + \frac{7}{2}y + 3} dx dy$



$y^2 - \frac{5}{2} = -\frac{5}{3}y - 1 + 5 \quad | \cdot 3$

$3y^2 - \frac{5}{2}y = -\frac{5}{3}y - 1 + 15$

$3y^2 - \frac{1}{2}y - 8 = 0$

$y_{1,2} = \frac{\frac{1}{2} \pm \sqrt{\frac{1}{4} - 16}}{6}$

$y_{1,2} = \frac{\frac{1}{2} \pm \sqrt{\frac{63}{4}}}{6}$

$y_1 \approx 2,7$

$y_2 \approx -2,54$

$\iint_D dx dy$

$D = \int_{-1}^{\frac{2}{3}} \int_{\frac{-1(y+1)^2-5}{2}}^{\frac{2}{3}y^2 + \frac{7}{2}y + 3} dx dy$

$D = \int_{-1}^{\frac{2}{3}} \left(\frac{5}{3}y - 1 \right) \cdot \left(-\frac{5}{3}y - 1 \right) dy$

$= \int_{-1}^{\frac{2}{3}} \left(\frac{25}{9}y^2 - \frac{10}{3}y + 1 \right) dy$

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b. $z = x^2 + y^2$ $z = 2$

$x^2 + y^2 - z = 0$

za $y=0$

$z = x^2 + y^2$

$x^2 = z$

$x^2 = z - y^2$

$x = \pm\sqrt{z}$

$x = \pm\sqrt{z - y^2}$

projeicija cilindrični koordinata

$$\begin{cases} x = r \cos \phi \\ y = r \sin \phi \\ z = z \end{cases}$$

$r^2 \cos^2 \phi + r^2 \sin^2 \phi - z = 0$

$r^2 - z = 0$

$r^2 = z$

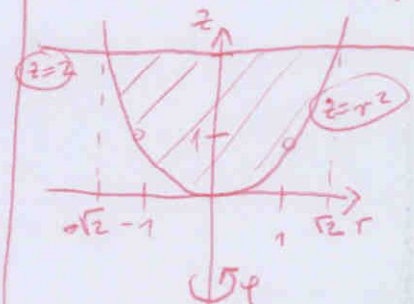
$r = \sqrt{z}$

$r = \pm\sqrt{z}$

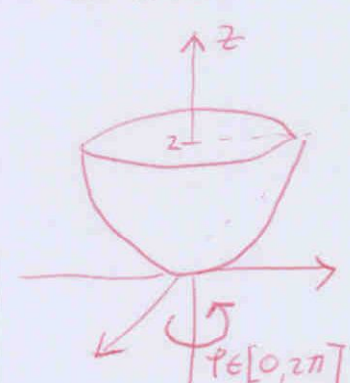
$z = r^2$

$z = 2$

SKICA U $z-r$ RAVNINI:



\Rightarrow 3D SKICA



$V = \int_0^2 \int_{-\sqrt{z}}^{\sqrt{z}} \int_{-\sqrt{z-y^2}}^{\sqrt{z-y^2}} r \, dr \, dy \, dz$

$V = \int_0^2 \int_{-\sqrt{z}}^{\sqrt{z}} \int_{-\sqrt{z-y^2}}^{\sqrt{z-y^2}} r \, dr \, dy \, dz$

