Primjer. Nađi koliko iznosi $f(2.5)$ ako f zadovoljava $dy = \left(1 + x + \frac{y}{1-x^2}\right) \mathrm{d}x$ i $y(0) = 0$.

$$y' = 1 + x + \frac{y}{1-x^2}$$

$$e^{a\cdot b} = (e^a)^b$$

~~SEPARILNA~~ LINEARNA ✓ ~~HOMOGENA~~

$$\left(e^{-\frac{1}{2}}\right)^{\ln||}$$

$$\left(e^{\ln()}\right)^{-\frac{1}{2}}$$

$$\longrightarrow y' + \frac{y}{1-x^2} = 1 + x$$

$$y' + \frac{1}{1-x^2} \cdot y = 1 + x$$

ONA D.S. PA RIJEŠI: $y' + \frac{1}{1-x^2} \cdot y = 0 \Rightarrow y' = -\frac{y}{1-x^2} \Rightarrow \int \frac{dy}{y} = \int \frac{-dx}{1-x^2}$ $\quad / \cdot \frac{dx}{y}$

$$\ln|y| = -\frac{1}{2}\ln\left|\frac{1+x}{1-x}\right| + C \quad / e^{\square}$$

$$y(x) = C\, e^{-\frac{1}{2}\ln\left|\frac{1+x}{1-x}\right|} = C\left(\frac{1+x}{1-x}\right)^{-\frac{1}{2}} = C \cdot \sqrt{\frac{1-x}{1+x}}$$

VARIRATI KONSTANTU $\quad C = u(x)$

TRAŽIMO RJEŠENJE POLAZNE JEDN. U OBLIKU $y(x) = u(x) \cdot \sqrt{\frac{1-x}{1+x}}$

$$\sqrt{\frac{1-x}{1+x}}' = \frac{1}{2}\,\frac{\frac{-(1+x)-(1-x)}{(1+x)^2}}{\sqrt{\frac{1-x}{1+x}}}$$

$$u'(x) \cdot \sqrt{\frac{1-x}{1+x}} \cancel{- u(x) \cdot \frac{\sqrt{1+x}}{\sqrt{1-x}\,(1+x)^2}} + \cancel{\frac{1}{1-x^2} \cdot u(x)\sqrt{\frac{1-x}{1+x}}} = 1 + x$$

$$\frac{\sqrt{1+x}}{\sqrt{1-x}\,(1+x)^2} \overset{?\checkmark}{=} \frac{\sqrt{1-x}}{\sqrt{1+x} \cdot (1-x)(1+x)}$$

$$(1-x)^{-\frac{1}{2}}(1+x)^{-\frac{3}{2}} \overset{?\checkmark}{=} (1-x)^{-\frac{1}{2}}(1+x)^{-\frac{3}{2}}$$

✓ ✓

$$u'(x) \cdot \sqrt{\frac{1-x}{1+x}} = 1 + x$$

$$u'(x) = (1+x)\sqrt{\frac{1+x}{1-x}} = \frac{(1+x)^{\frac{3}{2}}}{(1-x)^{\frac{1}{2}}}$$

$$u(x) = \int \frac{(1+x)^{\frac{3}{2}}}{\sqrt{1-x}}\,dx = \left\{\begin{array}{l} t = 1-x \Rightarrow x = 1-t \\ dt = -dx \end{array}\right\} =$$

$$u(x) = -\int \frac{(2-t)^{\frac{3}{2}}}{\sqrt{t}}\,dt = \int \sqrt{\frac{(2-t)^3}{t}}\,dt \ \ldots =$$