Primjer. Nađi opće rješenje jednadžbe $(x-y)y\,\mathrm{d}x - x^2\mathrm{d}y = 0$. $\quad / \cdot \frac{1}{dx}$

Zapamti: $y' = \frac{\mathrm{d}y}{\mathrm{d}x}$.

$$\underbrace{(x-y)y - x^2 y' = 0}$$

$y \quad y \quad y'$

Ključ rješavanja je prepoznati tip!

~~SEPARIRANA~~

$$-x^2 y' = -(x-y)y$$

$$y' = \frac{x-y}{x^2}\cdot y$$

~~LINEARNA~~

$$y' - \boxed{\frac{(x-y)}{x^2}}\, y = 0$$

HOMOGENA ✓

$$y' = \left(\frac{1}{x} - \frac{y}{x^2}\right)\cdot y = \frac{y}{x} - \left(\frac{y}{x}\right)^2$$

$$\frac{y}{x} = u \qquad \boxed{y = u\cdot x}$$

$$y' = u'\cdot x + u$$

$$\int x^\alpha = \frac{x^{\alpha+1}}{\alpha+1}$$

$$\int u^{-2} = \frac{u^{-1}}{-1}$$

$$u'\cdot x + \cancel{u} = \cancel{u} - u^2$$

SEPARIRANA ODJ: $\displaystyle\int \frac{du}{u^2} = \int \frac{-dx}{x} = -\ln|x| + C$

$$\frac{-1}{u(x)} = -\ln|x| + C \Rightarrow u(x) = \frac{-1}{C - \ln|x|}$$

$$\boxed{y(x) = \frac{-x}{C - \ln|x|}}$$

$$y'(x) = \frac{-(C-\ln|x|) + x\left(0 - \frac{1}{x}\right)}{(C-\ln|x|)^2} = \frac{-(C-\ln|x|) - 1}{(C-\ln|x|)^2}$$

$$\left(x - \frac{-x}{C-\ln|x|}\right)\frac{-x}{C-\ln|x|} - x^2\,\frac{-C+\ln|x| - 1}{(C-\ln|x|)^2} = \frac{-x^2}{C-\ln|x|} - \cancel{\frac{x^2}{(C-\ln|x|)^2}} + \frac{Cx^2 - x^2\ln|x| + \cancel{x^2}}{(C-\ln|x|)^2}$$

Uvijek je dobro provjeriti rješenje!

$$= \frac{-x^2(\cancel{C - \ln|x|}) - x^2\cancel{\ln|x|} + \cancel{Cx^2}}{(C-\ln|x|)} = 0$$